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Calculation of Fuel-Optimal Aircraft Flight Profile

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Abstract

Sedan världens första konventionella flygplan lyfte 1914 har flygindustrin förbättrats konstant under de 104 åren sedan dess. År 2017 transporterades över 4.1 miljarder passagerare med ungefär 36.8 miljoner flights av världens alla flygbolag. Statistik visar på att ungefär 2% av människoskapad emission av koldioxid kommer från flygindustrin.

För att värna om miljön och reducera emission av koldioxid så blir det därmed viktigt att reducera konsumtionen av flygbränsle. Flygplanstillverkare har redan tillämpat många metoder för att spara flygbränsle, såsom förbättringar av aerodynamik för flygplan och förbättring av motoreffektivitet, samt i senare år att applicera kompositmaterial för att reducera flygplanens vikt. För flygbolag så är en lämplig och ekonomisk färdplan hjälpsam för att reducera konsumtion av flygbränsle. Utöver konsumtion av flygbränsle så är även tid en lika viktig faktor för flygbolag.

Denna avhandling tar ett flygledningsperspektiv och etablerar en numerisk simulering baserad på dynamic programming för att beräkna den mest optimala vertikala flygbanan som följer ATC:s (Air Traffic Control) begränsningar och som använder sig av högupplöst väderdata.

Abstract

Since the world's first fixed-wing scheduled aircraft took-off in 1914, with the development on commercial aircraft, the aviation industry has improved constantly in the following 104 years [1]. In 2017, over 4.1 billion of passengers were carried by about 36.8 million of flights by the world's airlines. Statistic number also shows that about 2% of human-induced carbon dioxide emission should be responsible by the aviation industry [2].

To protect the environment and reduce carbon dioxide emission, one important way is to reduce jet fuel consumption. Aircraft manufacturers has already employed many fuel saving methods such as improving aircraft aerodynamics and engine efficiency, and apply composite materials to reduce aircraft weight in recent years. For airlines, a suitable and economical flight plan is helpful to reduce fuel consumption. However, in addition to fuel consumption, time is another equally important factor for airlines at the same time.

This thesis starts from the flight management point of view, based on dynamic programming method, establish a numerical simulation to calculate the most optimal vertical flight trajectory under ATC (Air Traffic Control) constrains and up-to-date high-resolution weather information.

Acknowledgements

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Contents

1 Introduction

The FMC (Flight Management Computer) is the brain if the aircraft, it calculates and tells the aircraft how to fly. Due to the calculation ability limitation of FMC, without any hardware upgrade, it can only optimize vertical flight route within a certain upcoming range rather than an overall optimization from the departure airport to the destination airport.

According to AVTECH's demand, the optimal trajectory calculation problem is simplified and modeled as a 2D space searching problem. The simulation supposed to go through all possible solutions within the region that divided by flight levels and weather points and picks the best vertical trajectory. This simulation aims on providing a low cost optimization solution which can give pilot enough information to control the aircraft to cruise at most efficient flight level without any upgrade on existing hardware in cockpit. It based on a deep understanding of FMC calculation algorithm and does not cause any disturb on embedded flight dynamics nor any risk on flight control.

Many optimization algorithms has been applied to solve space searching problems in order to reduce aircraft fuel consumption and emission. European researchers published several papers include using genetic selection algorithm, ant colony optimization algorithm, beam search algorithm as well as many other algorithms to reduce flight cost [3, 4, 5]. Inspired by a paper implemented Floyd-Warshall algorithm which is a dynamic programming algorithm that finds the shortest path in a weighted graph, after discussed with supervisors in both AVTECH and KTH, the classical dynamic programming algorithm was selected for this simulation [6]. This classical method is efficient to find global optimal value and it has been widely applied in similar researches as well. Y. Miyamoto and other researchers from Kyushu University in Japan started from the optimal control point of view, built a 4D flight trajectory optimization model based on a dynamic programming approach in recent years [7, 8]. They set aircraft heading angle, flight path angle and engine thrust as control variables to calculate an optimal 4D trajectory. Even back in 1997, P. Hagelauer and F. Mora-Camino from France applied dynamic programming method to optimize aircraft 4D trajectory [9]. Except focus on optimizing both horizontal and vertical flight trajectory, they introduced soft computing techniques and reduce searching space by additional constrains in order to improve computing time by FMC. Nowadays, it would be unnecessary to perform all calculations by FMC itself. Instead, the optimization could be done by remote servers and results are transmitted to an EFB (Electronic Flight Bag) device in the cockpit.

The simulation introduced in this thesis run through all en-route phases including climb, cruise and descent. It only performs calculation for altitude higher than $3,000$ ft. Below $3,000$ ft, with more regulations and constrains, aircraft are compelled to follow the take-off and landing procedure (LTO cycle) given by local airport control tower. Besides, many uncertainties depends on weather condition, local terrains and airport operation should be considered at low altitude. It would be complicated and unnecessary to take this part into the simulation. In climb and descent phase, the aircraft fly at speeds calculated by FMC under ECON mode.

The dynamic programming (DP) method is only applied to optimize vertical flight route for the cruise phase.

2 Aircraft and Engines

2.1 Aircraft introduction

The simulation is performed with Boeing 737 MAX 8 equipped with two LEAP-1B turbofan engines. The Boeing 737 MAX is an American narrow-body aircraft series designed and produced by Boeing Commercial Airplanes as the fourth generation of the Boeing 737. Compares with previous generations, besides some modifications in the air-frame, B-737 MAX 8 has more powerful engines and split-tip wing-lets which improve wing aerodynamics notably [10]. Specifications of this type of aircraft is given in table 1.

Figure 1: B-737 MAX 8

Length	129 ft 8 in / 39.52 m
Height	40 ft 4 in / 12.3 m
Wing Span	117 ft 10 in / 35.92 m
MOTW	$181,200$ lb / $82,191$ kg
MLW	$152,800$ lb / $69,309$ kg
ZFW	$145,400$ lb / $65,952$ kg
Fuel Capacity	$25,817$ L
Cruise Speed Mach	0.79
VMO	340 knots
MMO	0.82
Ceiling Altitude	$41,000$ ft

Table 1: B-737 MAX 8 specifications

This engine LEAP-1B has high by-pass ratio that can supply maximum $28,000$ lbf of thrust (maximum takeoff thrust) to the aircraft. It offers 737 MAX operators exceptional technical, economic and environmental performance, with a 15% reduction in fuel consumption and CO_2 emissions versus current engines, a 50% cut in NO_x emissions, and compliance with the most stringent noise standards [11].

3 Airlines Operating Logistics

3.1 Flight Phase Overview

A complete commercial flight mainly contains three phases: climb, cruise and descent. The following figure 2 shows the vertical profile for those three phases [12]. The speed for aircraft lower than $3,000$ ft for both climb and descent phase in the figure are expressed as KIAS (Indicated Airspeed in Knots). In Boeing's aircraft manual, it is more common to use the KCAS (Calibrated Airspeed in Knots) instead of KIAS. The different between IAS and CAS is introduced in appendix A.

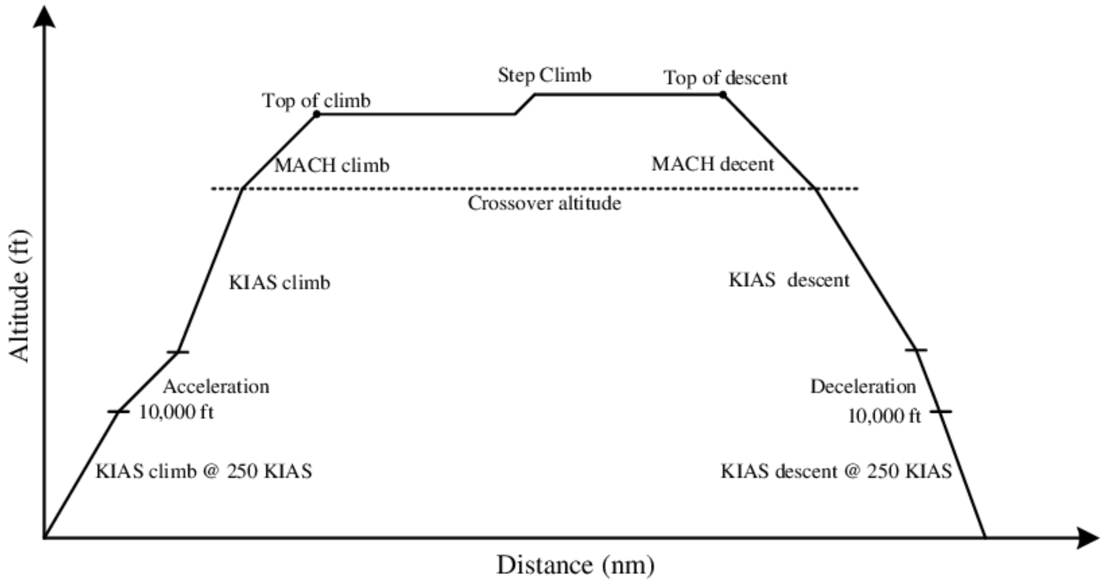


Figure 2: Flight phases

3.1.1 Climb Phase

When the aircraft just left from the airport control area and starts to perform an initial climb from $3,000$ ft to $10,000$ ft, it is not allowed to climb faster than 250 KCAS due to regulations. Crosses $10,000$ ft, the aircraft accelerates to a required climb speed in a short time and continuous climb with the new speed until the crossover altitude. At the moment that the aircraft reaches the crossover altitude, the aircraft automatically switches to fly at Mach mode. The climb speed remains a constant Mach number between crossover altitude and TOC (Top of Climb) altitude.

This switch of mode at crossover altitude is due to the limitation of aircraft maximum dynamic pressure with a certain buffet. If the aircraft fly with constant KCAS instead of constant Mach at high altitude, its corresponding TAS (True Airspeed) and dynamic pressure will be too high which may cause damage on aircraft structure. But a constant Mach number at high altitude can guarantee both TAS and dynamic pressure decrease while altitude increase. The definition and calculation method for crossover altitude is introduced in appendix C [14].

3.1.2 Cruise Phase

Cruise phase is generally defined as the phase between TOC and TOD (Top of Descent). For a intercontinental or long distance flight, this phase normally consumes most of the time and fuel of the entire flight. To reduce fuel consumption, pilots intend to cruise at higher altitude where has lower air density and higher engine efficiency. Whether to perform step climbs or descents is usually decided by pilots with en-route weather information and approved by ATC. For a domestic or regional flight, the aircraft has a probability to be scheduled to cruise at a constant flight level or perform as few altitude changes as possible.

The cruise Mach is interpolated by FMC with temperature and wind as basic inputs. Furthermore, there exist a threshold CAS value in case of extreme weather condition results in an earlier descend before the descent mode has been triggered. Assume an aircraft descends to a lower altitude to avoid severe turbulence, while the CAS converted from current cruise Mach increase to the same value as the threshold CAS, the FMC should be able to switch to a mode that cruise with a constant CAS.

3.1.3 Descent Phase

The descent phase is almost a reverse process of the climb phase. The FMC estimates an approximate descent region based on rule of 3 (in appendix D) with the TOD altitude selected by pilots or airlines [15]. When the aircraft enters into the descent region, it triggers the descent mode and the aircraft starts to descend with a constant Mach number. Similarly, the aircraft switches to the constant KCAS descent mode at crossover altitude which avoids the problem of structure failure. Due to the same regulation, the descent speed should be lower than 250 KCAS below $10,000$ ft.

3.2 Cost Calculation

The cost for a flight comes from many different aspects. Statistic data shows that most airlines spend more than 20% of the total operating on jet fuel. Another important part of the cost is the crew labor cost which usually depends on working time. Besides, equipment maintenance cost, airport fee and other costs are commonly known as fixed costs. The total cost per flight in US dollar can be expressed as,

$$C = \frac{C_f{\cdot}F}{100} + C_t{\cdot}T + C_{fix} \tag{1}$$

where C_f is the unit fuel related cost in cents per pound of fuel and C_t is the unit time related cost in dollars per hour. The number 100 in equation 1 is due to the conversion between dollar and cents. F and T represent for fuel and time consumption for a flight respectively. The fixed costs term C_{fix} is a constant which does not various with fuel consumption nor flight time which could be ignored in following calculations.

3.3 Cost Index

Getting rid of the last term C_{fix} in equation 1, dividing both side of the remaining equation by C_f, after rearranging terms, the cost function (CF) is defined as,

$$CF = \frac{100 \cdot C}{C_f} = F + 100 \cdot CI \cdot T \tag{2}$$

where CI is the cost index which defined as the ratio between unit time cost C_t and unit fuel cost C_f.

$$CI = \frac{C_t}{C_f} \tag{3}$$

From its definition, the cost index measure the importance between time and fuel consumption. Theoretically, the value of cost index, as a ratio, can goes to positive infinity. But in real FMC, the maximum cost index could be set as a big number or even 999. For B-737 MAX 8, its maximum cost index is 800.

The cost index usually decided by airline flight management department according to their operating philosophy. While making the flight plan, the flight management department has to consider factors including aircraft type, engine type, crew configuration, flight route and statistic en-route weather data to choose a proper cost index for a specific flight.

At minimum cost index ($CI = 0$), the fuel cost is more important than time cost and the aircraft should cruise at lower speed with lower fuel consumption. To make it easier to understand, here introduce the concept of fuel mileage which is known as the aircraft average fuel consumption for cruising $1,000$ nautical miles. With the minimum cost index, the aircraft has maximum fuel mileage which gives the maximum cruise distance. The cruise speed calculated under minimum cost index is known as MRC (Maximum Range Cruise) speed which only describes a theoretical approach of the aircraft cruising ability. In practical, the LRC (Long Range Cruise) speed is commonly employed which is defined as a speed slighter higher that MRC speed that only results in 1% decrease of the fuel mileage. Graphically, the LRC speed fuel mileage equals to 99% of the MRC speed fuel mileage as in figure 3 [13].

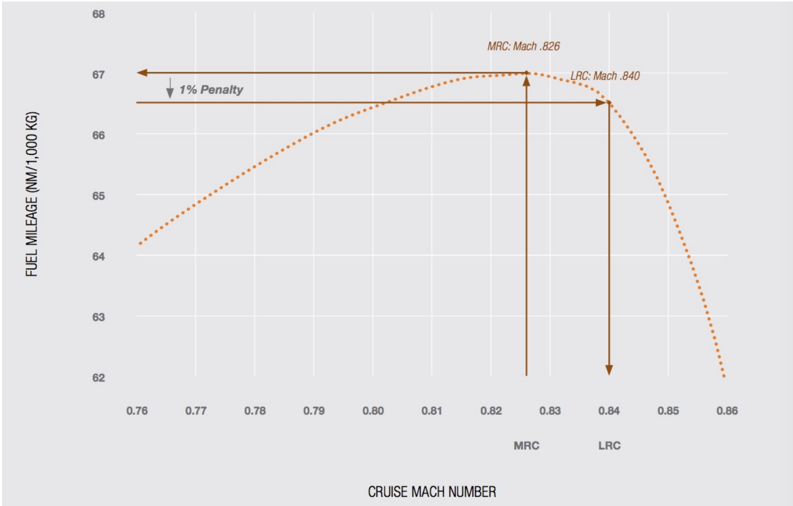
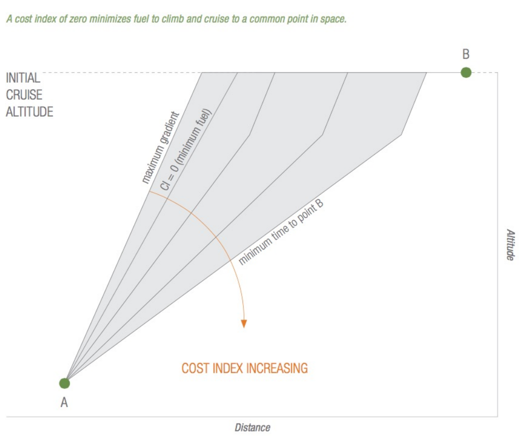


Figure 3: LRC and MRC

Contrarily, the time is a more important factor at maximum cost index. Under this cost index setting, the aircraft cruise with maximum cruise speed. Even though the aircraft cruises with relatively high fuel consumption, it can save a lot of time in return. High cost index maybe not typically preferred by commercial airlines, but it could be popular for aircraft run by express delivery services company which values time a lot.

Except affecting on cruise phase, the cost index also has influence on climb and descent phase. Figure 4 shows climb and descent profile with different cost index [13]. With a lower cost index, the aircraft climbs with a larger climb angle and descends with a shallower descent angle.



(a) Climb

(b) Descent

Figure 4: Climb and descent profile for different cost index

3.4 ECON Mode

Knowing that fuel cost is not the only direct cost related to a flight, the economical speed which includes some trade-offs between flight time and fuel consumption can be interpolated by FMC. The mode that FMC determines the economical speed is called ECON mode. This mode is allowed to apply for all three en-route phases and most airlines use ECON speed as VNAV (Vertical Navigation) speed during cruise phase.

During climb phase, the economical climb CAS for aircraft above $10,000$ ft and below crossover altitude is interpolated directly from Boeing's aircraft operating charts by function,

$$CAS_{climb} = f_{climb}(W_{TOC}, CI) \tag{4}$$

The aircraft weight W_{TOC} in equation 4 is the estimated aircraft weight at TOC point. The climb CAS should never exceed maximum climb speed constrained by VNAV limitation. For B-737 MAX 8, it can not climb with CAS higher than 335 knots at all altitudes. The climb CAS read by equation 4 is under the assumption of standard day temperature and zero wind condition. A correction of this speed based on real wind and temperature at TOC is needed.

Similarly, the economical descent CAS in the same altitude range can be interpolated without any correction by taking aircraft weight at TOD and cost index as inputs. The economical descent speed is limited to be no faster than 330 knots to facilitate descent path stability.

$$CAS_{descent} = f_{descent}(W_{TOD}, CI) \tag{5}$$

In Boeing's aircraft operating manual, the exact economical cruise Mach number is given as a function of aircraft weight and cost index for pressure altitude vary from $10,000$ ft to $41,000$ ft for standard day and zero wind condition. Obviously, it would be difficult for FMC to interpolate data from those tables. As a solution, the cost index is not directly used to interpolate economical cruise Mach number in FMC. Instead , the CCI (Corrected Cost Index) is employed which is defined as an atmospheric temperature and pressure correction of cost index,

$$CCI = \frac{CI}{\delta\theta^x} \tag{6}$$

where $\delta = P/P_0$ is atmospheric pressure ratio and $\theta = T/T_0$ is atmospheric temperature ratio. T_0 and P_0 represent temperature and pressure at sea level under ISA condition. The exponent x is unique for different aircraft type and engine engaged. For B-737 MAX 8 equipped with two LEAP-1B engines, the exponent $x = 0.63$.

A further correction on CCI based on real wind is also required, normally called the wind-adjusted cost index value ($CCIW$).

$$CCIW = CCI + CCF \cdot M_{ZW} \cdot \frac{V_T - V_G}{V_G} \tag{7}$$

V_T and V_G in equation 7 represent TAS (True Airspeed) and GS (Ground Speed) respectively. The definition of those speeds can be found in appendix A and the calculation method will be introduced in chapter 6.2.2. The corrected cost function value (CCF) and zero wind economical Mach number (M_{ZW}) are both interpolated from tables with a known CCI and W/δ.

$$M_{ZW} = f_{cruise}(W/\delta, CCI) \tag{8}$$

$$CCF = f_{CCF}(W/\delta, CCI) \tag{9}$$

Recall equation 8, put $CCIW$ instead of CCI,

$$M_{cruise} = f_{cruise}(W/\delta, CCIW) \tag{10}$$

The Mach number read by equation 10 is the economical cruise Mach number. It has been optimized and adjusted depends on temperature and wind condition.

Notice there are two special cases for reading economical cruise speed. The first one is that if the aircraft is cruising at minimum cost index, the FMC will not slow down the aircraft even it has benefit tailwind. The other one is that if the aircraft is cruising with maximum cost index, the economical cruise Mach will not be affected by wind nor temperature. Those special cases are decided due to safety reasons.

4 Dynamic Programming (DP)

Dynamic programming (DP) is a mathematical tool developed by Richard Bellman in the 1950s to solve multi-step decision process problems. It has been applied in solving a wide range of decision process problems from the field of engineering to economics. The main idea of this method is to break a sophisticated problem into several simple sub-problems in a recursive manner, then find the local optimal solution for those sub-problems forwardly (top-down) or backwardly (bottom-up). Notice that each sub-problem only solved once, and its solution will be stored and used for solving next sub-problem. The substructure that achieves the global optimal solution is also known as the optimal substructure. Some important features of dynamic programming will be introduced with the following diagram.

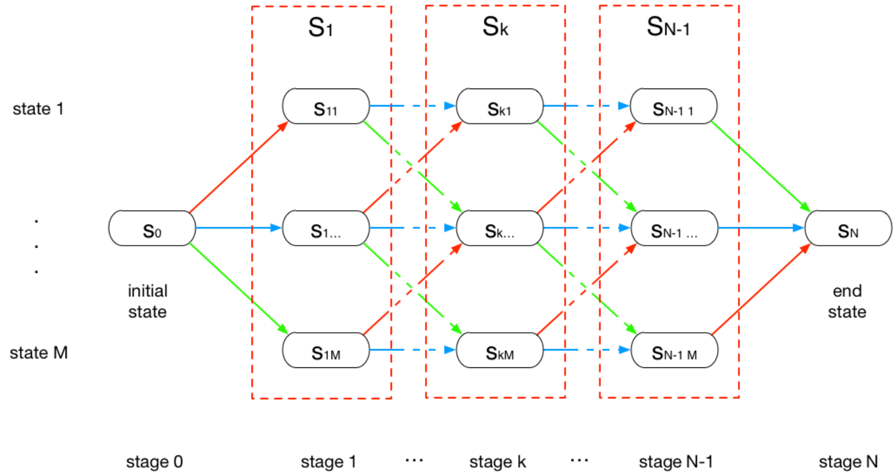


Figure 5: General dynamic programming diagram

4.1 Stages and States

From a mathematical optimization point of view, a sophisticated problem can be divided naturally into numbered sub-problems under time orders or space characteristics. Each sub-problem is also known as a stage. On each stage, there exists finite states which describe different status.

In figure 5, a problem has been divided into totally N stages, and there are in total M available states from stage 1 to stage $N-1$. The only state on stage 0 is also known as initial state. Similarly, the only state on stage N is called as end state. S_k in the figure represents the set of all admissible states on stage k.

$$S_k = \{s_{k1}, s_{k2}, ..., s_{ki}, ..., s_{kM}\} \tag{11}$$

4.2 Decisions and Policy

Once all states on the previous stage are decided, it is time to make the choice about how to reach to the next stage. Start from the i-th state on the stage k, a set of all feasible decisions on this state is defined as $D_{k_i}(s_{k_i})$. And the real decision is only allowed to be one of the decision within this set. which means,

$$d_{ki} \in D_{ki}(s_{ki}) \tag{12}$$

Generally, having a set of all feasible decisions on stage k, the decision to move from stage k to $k+1$ can be represented as,

$$d_k \in D_k(s_k) \tag{13}$$

Where s_k in equation 12 means a state on stage k, and it satisfies $s_k \in S_k$. Finally, a set of all decisions on each stage consist a policy for the problem.

$$P = \{d_0, d_1, d_2, ..., d_{N-1}\} \tag{14}$$

4.3 State Transition Equation and Objective Equation

When the state s_k and decision d_k on stage k are decided, the state on the next stage can be calculate as well. The function describe the relationship between s_k, s_{k+1} and d_k is called state transition equation.

$$s_{k+1} = t_k(s_k, d_k) \tag{15}$$

Using V_k for objective equation at stage k, the contribution to the objective equation by stage k under s_k and d_k is,

$$\Delta V_k = v_k(s_k, d_k) \tag{16}$$

Clearly, the objective equation at stage $k+1$ is,

$$V_{k+1} = V_k + \Delta V_k \tag{17}$$

4.4 Principle of Optimality and Optimal Policy

The principle of optimality states that an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision [16]. i.e., status and decisions made on stage before k has no influence on the contribution to the objective function by remaining stages after k, and vice verse. For a backward dynamic programming, the contribution to the objective equation from the k-th stage to the last stage is,

$$\Delta V_{k\to N} = \sum_{k}^{N} v_k(s_k, d_k) \tag{18}$$

Now let $v_k^*(s_k)$ denotes the optimal value of the equation 18, in the mathematical format,

$$v_k^*(s_k) = \sum_{k}^{N} \underset{d_k \in D_k(s_k)}{opt} v_k(s_k, d_k) \tag{19}$$

The set of decisions $P_{k\to N}^* = \{d_k^*, ..., d_N^*\}$ guarantee the value of objective function reach to $v_k^*(s_k)$ is called the optimal policy from the k-th stage to the last stage. It should satisfy that on each stage $d_k^* \in D_k(s_k)$. The overall optimal policy for the problem is a set of optimal decision on each stage.

$$P^* = \{d_0^*, d_1^*, d_2^*, ..., d_{N-1}^*\} \tag{20}$$

4.5 Optimization Procedures

The problem solved by dynamic programming is usually processed backwardly which has been described by equations above. Even though the theory and equations look very complicated, but the logistics and procedures for solving a problem are simple and straight forward. It is better to demonstrate its procedures with the following problem shown in figure 6. The problem aims is to find the path with minimum cost from s_0 to s_3 where numbers between two states represent costs for sub-paths.

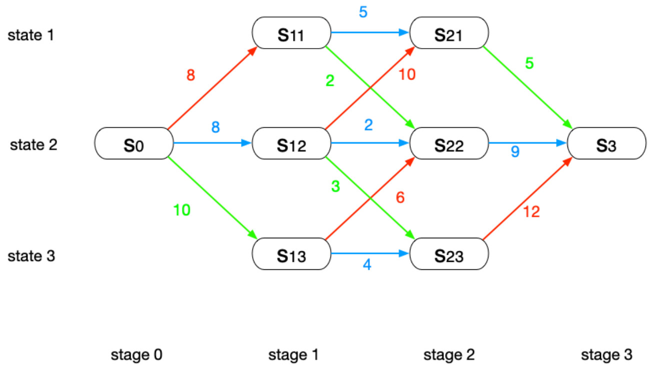


Figure 6: An example for solving problem by dynamic programming

Solve the problem backwardly means that the breakthrough point is the end state s_3. In the figure, Stage 2 is the stage before the last stage and there are three states on stage 2 connecting to s_3. In other words, there are three available choices to move from stage 2 to stage 3. Now take the cost as objective equation and it has zero value on s_3. Recall equation 18, the contribution to the objective equation from each state on stage 2 to stage 3 are,

$$\Delta V_{21\to 3} = 5 \qquad \Delta V_{22\to 3} = 9 \qquad \Delta V_{23\to 3} = 12 \tag{21}$$

Now move to the first state on stage 1, there are two available choices to move from s_{11} to stage 2. Recall equation 19, the optimal value of the objective equation for moving from s_{11} to s_3 is,

$$\Delta V_{11\to 3} = v_{11}^*(s_{11}) = min(5 + \Delta V_{21\to 3}, 2 + \Delta V_{22\to 3}) = min(10, 11) = 10 \tag{22}$$

thus the optimal decision for moving from s_{11} to s_3 is $s_{11}{\to}s_{21}{\to}s_3$. Similarly,

$$\Delta V_{12\to 3} = v_{12}^*(s_{12}) = 11 \qquad \Delta V_{13\to 3} = v_{13}^*(s_{13}) = 15 \tag{23}$$

and optimal path for those two are $s_{12}{\to}s_{22}{\to}s_3$ and s_3 is $s_{13}{\to}s_{22}{\to}s_3$ respectively. The last step is optimize the route from stage 0 to stage 1. Similar to equation 22,

$$v_0^*(s_0) = min(8 + \Delta V_{11\to3}, 8 + \Delta V_{12\to3}, 10 + \Delta V_{13\to3}) = min(18, 19, 25) = 18 \quad (24)$$

The minimum cost for moving from stage 0 to stage 3 is 18. And the optimal policy to achieve the minimum cost is $s_0 \to s_{11} \to s_{21} \to s_3$ which is also shown with yellow dash line in figure 7.

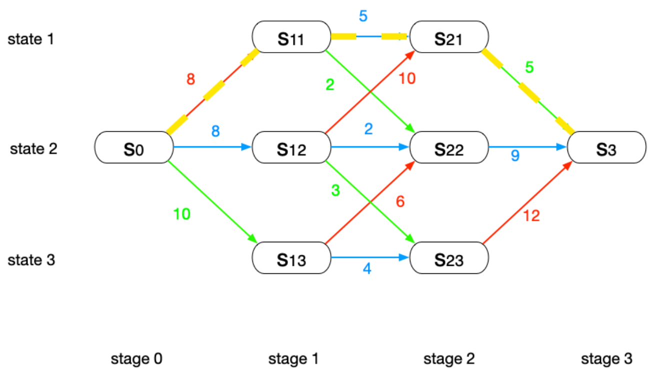


Figure 7: Optimal policy

The procedure introduced above for solving problems with dynamic programming method is in the backward way. Obviously, the problem can be solved forwardly as well. The direction in dynamic programming is flexible and it depends on which state has already clearly defined.

4.6 Advantage and Disadvantage

Dynamic programming method is a useful and efficient method for solving multi-objective optimization problems. It is suitable for linear, non-linear, discrete or continue variables. The logistics of dynamic programming method is simple and straight forward. Compare with genetic selection method, DP can guarantee to gain the global optimal value for the objective equation. Compare to traversal method, it decreases calculation time dramatically.

On the other hand, since the problem has to be divided into multi-stages, the subproblems can only be solved in a series sequence which consumes memories for the calculation. But for the problem with many states on each stage, it is possible to calculate for different states on the same stage at same time in a parallel way to reduce calculation time.

5 Basic Models

5.1 Weather Model

The weather data is generated and supplied by Met Office. The Met Office use weather data collected from both aircraft and ground observation as inputs for its own weather prediction model. This model can be considered as a CFD simulation around the earth which can supply the latest weather forecast data for airlines and other users. The weather data predicted by the model updated every 6 hours with time indicated in UTC (Coordinated Universal Time). Compare with classical weather prediction model, except providing high quality weather data, this model also increase data resolution for which the shortest distance between two weather points is only 5.4 nmi (about 10 km).

With a predefined flight plan contains way points information and scheduled flight time, the ground distance between departure airport and destination airport is divided into finite weather points. The distance between two weather points is decided based on a trade-off between model precision and calculation time. Clearly, it is true that higher weather data resolution can improve model precision, but it also increases model calculation time exponentially. In the simulation, the distance between two weather points for a short flight is roughly 50 km which guarantees the model precision, at the same time, makes it possible to provide simulation results to end users within a tolerable waiting time. The estimated time of the aircraft to reach each weather point is calculated with an average flight speed. A similar estimation logistic has been applied in a paper by R. Botez and R. Patrón by using genetic algorithms to optimize flight trajectory so as to reduce the number of possible trajectories [3]. With weather point location and arrival time, the predicted weather data including wind speed V_W, headwind speed V_{HW} and temperature T from $3,000$ ft to $42,000$ ft are acquired from weather prediction model in Met Office.

5.2 Aircraft Model

As mentioned before, the simulation is based on a deep understanding of FMC logistics and it should never causes any disturb on flight dynamics. Thus, the aircraft model used here is a simple point-mass model. The angle of attack, engine thrust and other aircraft control variables are automatically calculated by FMC itself.

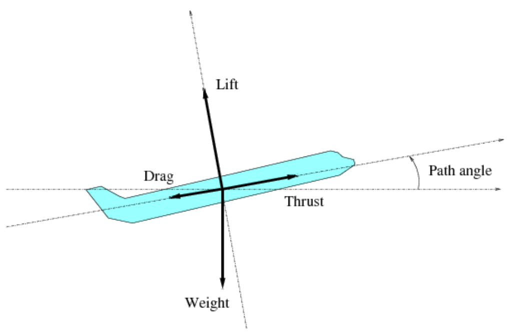


Figure 8: Aircraft point-mass model

6 Simulation Model

6.1 Assumptions and Simplifications

The real time for the aircraft arrivals to each weather point depends on previous choices on cruise altitude and cruise speed, thus it is different to the estimated arrival time calculated by average speed. Even though the weather data could be given in a dynamic way but it will dramatically increase weather data acquiring time and model complexity. In order to guarantee the model efficiency and precision at the same time, an assumption that upper air weather data does not change too much within the time difference is made.

Furthermore, wind and temperature are assumed to vary linearly between two nearby weather points. For the aircraft model, assume the aircraft gross weight remain constant while cruising from one stage to the next, all aircraft status are recalculated only when it reaches a new stage. Besides, at altitude $10,000$ ft for both climb and descent phase, the time for aircraft acceleration or deceleration is neglected.

6.2 Simulation Model Structure

The simulation model is modularized and described with the following diagram in figure 9.

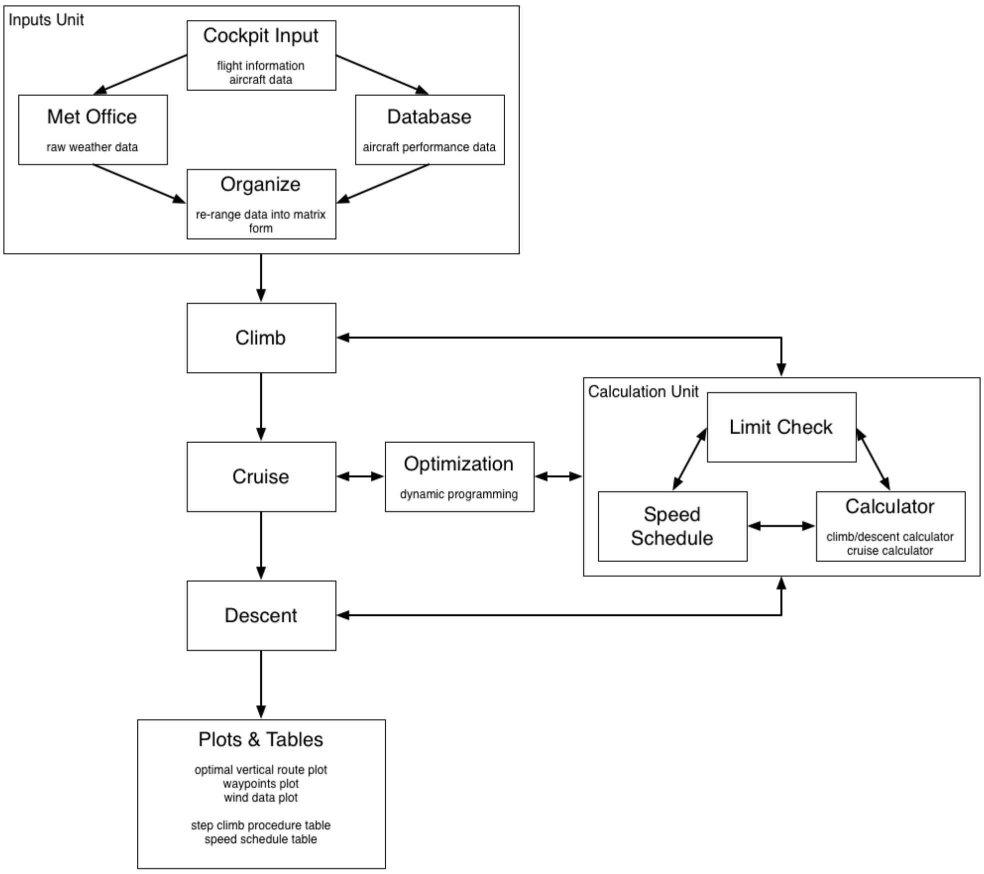


Figure 9: Simulation model diagram

In general, pilots insert aircraft data and flight path manually to FMC through CDU (Control Display Unit). With the flight route, it is possible to acquire en-route weather data from Met office. And with the aircraft data, the simulation can find fuel flow and rate of climb/descent tables for this specific aircraft in the database.

The simulation model run through climb, cruise and descent phase step by step. Dynamic programming optimization method is only applied for selecting optimal altitude in cruise phase. With all information mentioned above, the simulation is able to plot an optimal vertical route and generate tables for en-route speeds and activities. A more detailed introduction for different units and functions are described in following subsections.

6.2.1 Inputs Unit

In the inputs unit, the model collect all data that is necessary for the simulation. First, the simulation model reads flight information and aircraft data inserted by pilots. That information mainly includes flight plan, ATC constrains, aircraft type, gross weight and cost index. According to the flight plan, a weather data package will be acquired from Met office as mentioned in chapter 5.1. With aircraft data, a package for fuel flow and rate of climb/descent for this specific aircraft with the specified engine can be extracted from database. The raw weather data from Met office has its own format and it needed to be rearranged into a matrix form which is easier for interpolation.

6.2.2 Calculation Unit

The calculation unit is indispensable and it could be called by other functions thousands of times during the simulation. It contains limit check, speed schedule reading and calculator functions. The limit check function computes maximum certified altitude, buffet limit altitude and thrust limit altitude based on aircraft current status. If the aircraft tries to climb to an altitude that exceed any limit calculated above, the limit check function returns a negative index which denies the climb plan.

The speed schedule function is executed when the aircraft reaches the next stage, a new economical speed should be read and assigned for the stage after next. The economical speed reading logistics has been explained in chapter 3.4.

The most important part within the calculation unit is the calculator function. Firstly, the function calculates GS corresponding to the speed assigned in the previous stage by speed schedule reading function. A speed conversion for Mach number or CAS to TAS is necessary. Equations for speed conversion are given in appendix C [14]. The GS of the aircraft is a wind correction of TAS, graphically

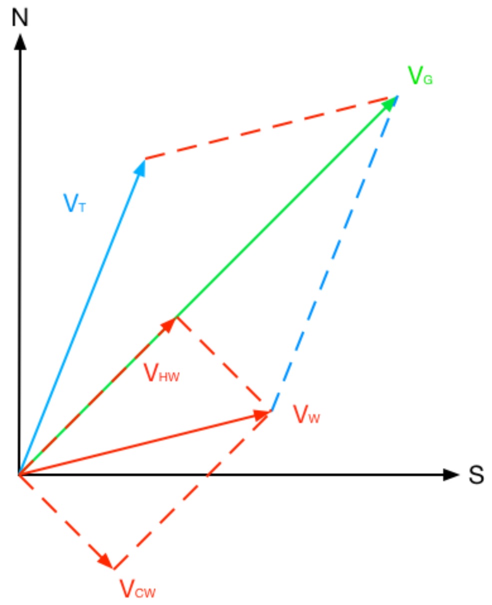


Figure 10: Wind correction to true airspeed

Figure 10 shows the speed triangle in the horizontal direction, which can be considered as a projection or top view of aircraft speed and wind speed. Using equation 25 to denote this speed relationship in the vector form,

$$\vec{\boldsymbol{V_G}} = \vec{\boldsymbol{V_T}} + \vec{\boldsymbol{V_W}} \tag{25}$$

Furthermore, decompose wind $\vec{\boldsymbol{V_W}}$ orthogonally into $\vec{\boldsymbol{V_{CW}}}$ and $\vec{\boldsymbol{V_{HW}}}$ terms. The term along the ground speed direction $\vec{\boldsymbol{V_{HW}}}$ is also known as headwind, and the term perpendicular to the ground speed direction denoted by $\vec{\boldsymbol{V_{CW}}}$ is called crosswind. Rewrite equation 25 in both vector and magnitude form,

$$\vec{\boldsymbol{V_G}} = (\vec{\boldsymbol{V_T}} + \vec{\boldsymbol{V_{CW}}}) + \vec{\boldsymbol{V_{HW}}} \tag{26}$$

$$V_G = \sqrt{V_T^2 - V_{CW}^2} + V_{HW} \tag{27}$$

Recall chapter 3.4, both V_G and V_T for computing $CCIW$ in equation 7 can be calculated by equations above.

For the climb phase, the rate of climb (RoC) on vertical direction has to be considered. It is calculated by FMC with current aircraft weight, altitude and engine thrust as inputs. The magnitude of ground speed can still be calculated by equation 27. But due to the presence of RoC, the horizontal true airspeed V_T rewrite as,

$$V_T = \sqrt{V_{TAS}^2 - RoC^2} \tag{28}$$

Where V_{TAS} is the magnitude of true airspeed. For descent, using rate of descent (RoD) instead of RoC, rewrite equation 28 as,

$$V_T = \sqrt{V_{TAS}^2 - RoD^2} \tag{29}$$

Secondly, knowing ground speed, aircraft status should be recalculated at a certain time. For cruise, all status are updated when the aircraft reaches a new weather point. The time for the aircraft cruise from current stage to the next stage is,

$$\Delta t = Range/V_G; \tag{30}$$

For climb and descent, aircraft status will be recalculated when the aircraft climb or descend for $1,000$ ft.

$$\Delta t = 1000/RoC; \tag{31}$$

$$\Delta t = 1000/RoD; \tag{32}$$

The RoC and RoD in equation 28, 29 31 and 32 were extracted from EUROCONTROL's BADA (Base of Aircraft Data) aircraft performance model [17]. The fuel flow (FF) data was pre-generated in the same way. Both rate of climb and fuel flow for B-737 MAX 8 are saved in a performance database which can be considered as a black box with following inputs and outputs.

<table>
<tr><th>Phase</th><th>Inputs</th><th>Outputs</th></tr>
<tr><td>Climb</td><td rowspan="3">Aircraft gross weight [lb]
Speed (in knots or Mach)
Altitude [ft]
ISA deviation [K]</td><td>Fuel flow [kg/min]
Rate of Climb [fpm]</td></tr>
<tr><td>Cruise</td><td>Fuel flow [kg/min]</td></tr>
<tr><td>Descent</td><td>Fuel flow [kg/min]
Rate of Descent [fpm]</td></tr>
</table>

Table 2: Inputs and outputs

With sufficient unit conversions, the fuel consumption within a given time period is,

$$\Delta W_f = FF \cdot \Delta t \tag{33}$$

Since the unit fuel cost depends on the contract between airlines and fuel suppliers, here define a equivalent fuel consumption instead of total cost. Recall the cost function equation 2, using the same definition, the equivalent fuel consumption for a given time period is,

$$\Delta W_{eq} = \Delta W_f + 100 \cdot CI \cdot \Delta t \tag{34}$$

At the end of each period, the aircraft reaches to a new stage. Recalculating the aircraft weight and use it to read the economical speed for the next stage. The new aircraft weight is,

$$W_{new} = W_{previous} - \Delta W_f \tag{35}$$

6.2.3 Climb

The climb function generates an altitude matrix from $3,000$ ft to the TOC with steps of $1,000$ ft. It passes this matrix and aircraft status to the calculation unit which returns an ECON climb speed schedule. The climb calculation procedure is denoted by figure 11. The result given by calculation unit is a package contains climb speed, aircraft status, and horizontal position for the aircraft at each step.

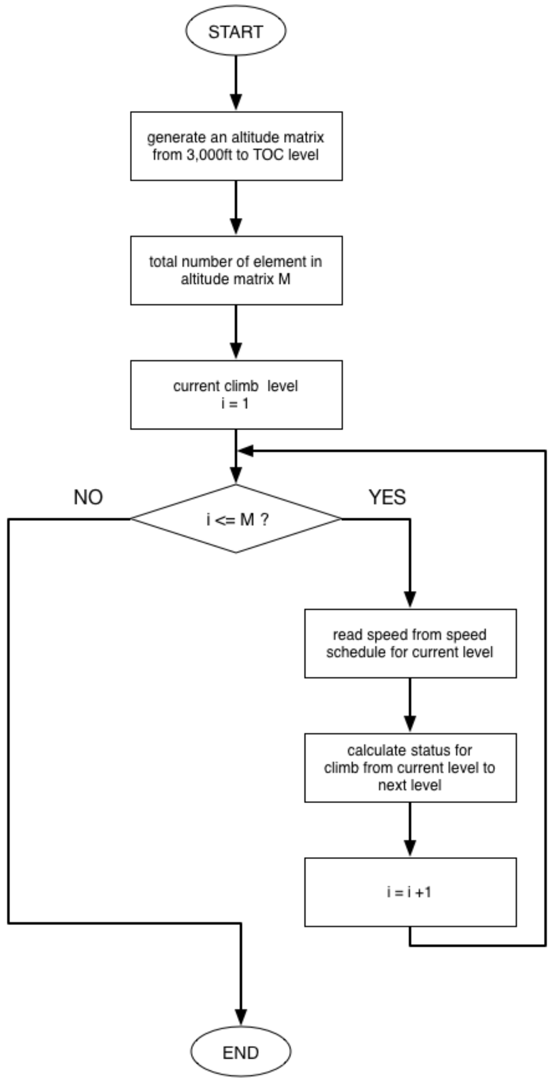


Figure 11: Diagram for climb calculation

6.2.4 Cruise and Optimization

When the aircraft reaches the predefined TOC altitude, the horizontal location of the TOC point is treated as stage 0 and the TOC altitude is considered as the initial state. The rest of stages of dynamic programming start from the first weather point appeared after TOC position to the destination airport, and states among each stage normally start from $24,000$ ft to $42,000$ ft with steps of $1,000$ ft. The range where dynamic programming algorithm is applied is displayed by figure 12. The red, blue and green arrows only denote that the aircraft may choose to climb, cruise or descend on each state from the first stage till the last stage.

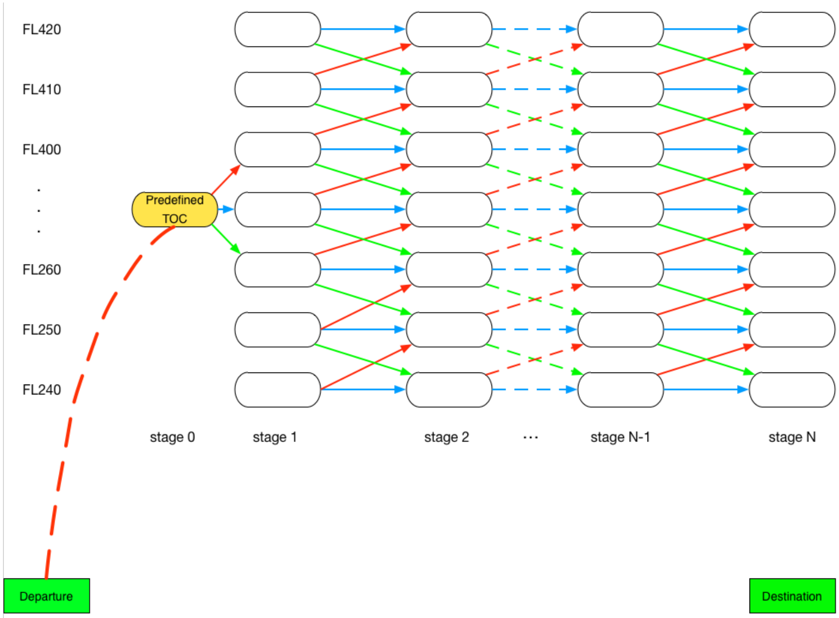


Figure 12: The en-route range with dynamic programming method applied

After all stages and states are defined, the calculation go through forwardly from the initial state until the last state of the last stage. This logistics can be demonstrate with the following flow chart in figure 13.

More specifically, in order to find the optimal trajectory to state i on stage k, the simulation should first screen out all available points that can potentially reach state i on stage k after climb, cruise or descent. The next step is to calculate aircraft status for reaching to this state via different trajectories from available points. Recall all status calculated by the calculation unit, the objective function for optimal policy decision making could be any of the aircraft status depending on which factor is more important for the airline. In order to consider both time and fuel cost at the same time, the equivalent fuel consumption W_{eq} calculated by equation 34 is set as the objective function in the simulation.

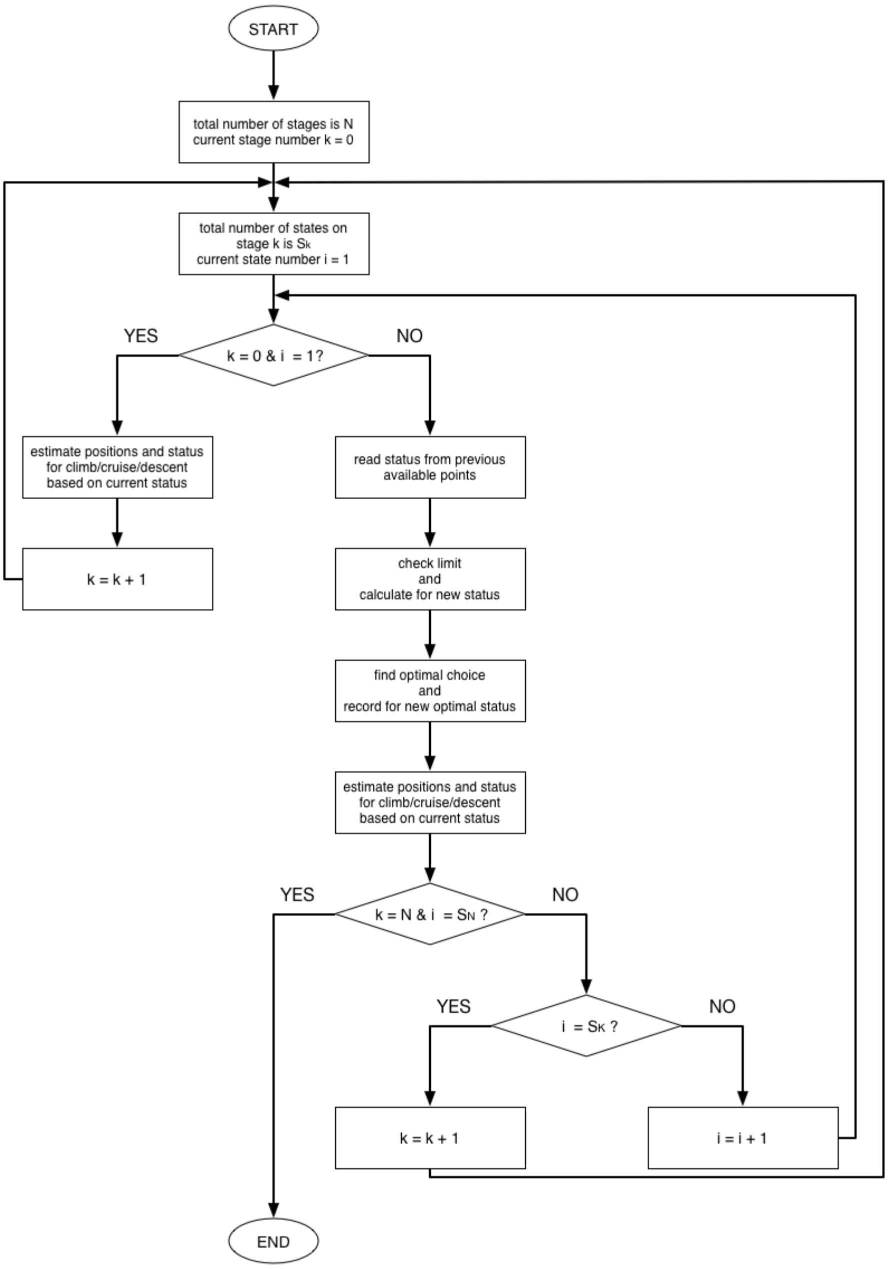


Figure 13: The flow chart for dynamic programming during cruise phase

Notice all previous available points that can potentially reach to state i on stage k does not equal to all states on stage $k-1$. As shown in figure 14, when the simulation starts to check state 4 on stage k, it has to collect all points in previous stages that can potentially reach to this state by climb, cruise or descent. Those points may come from stage $k-1$, $k-2$ or even further before stages. The bow shape distribution of those previous available points is due to aircraft climb and descent ability.

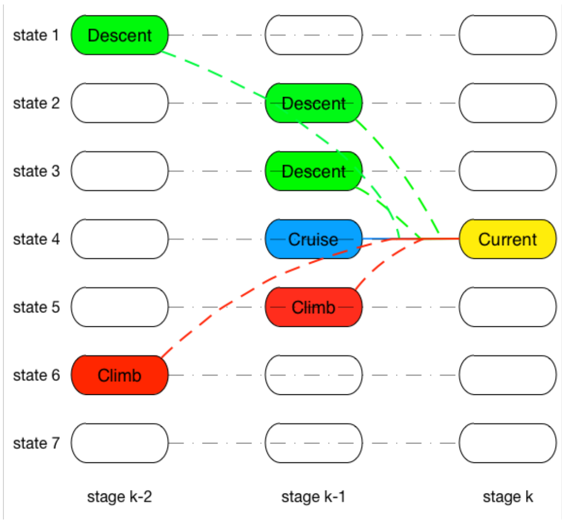


Figure 14: Available points in previous stages

Those available points are pre-calculated and selected by position estimation function. This function perform an estimation of climb, descent and cruise from the status defined point. As shown in figure 15, with a defined current status, the aircraft can climb, cruise or descend from the current location. As shown in the figure, the aircraft can move from state 4 on stage k to state 2 on stage $k+2$ by climb and a partially cruise. This partially cruise range is called remaining cruise distance in the simulation and its value should smaller than the horizontal distance between two nearby weather points. Due to the aircraft operating ability, some of the points can be never reached. The position estimation function is called every time when the aircraft reach to a certain state.

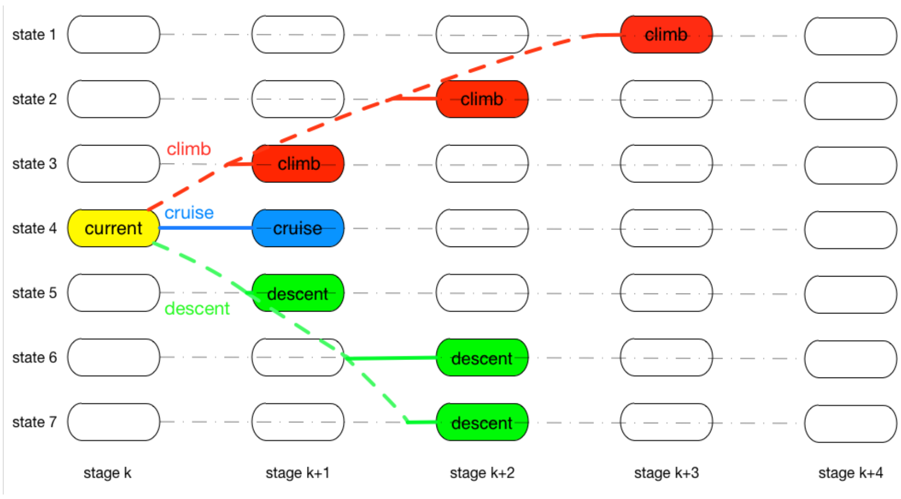


Figure 15: Position estimation

When the dynamic optimization solver end up at the last state of last stage, it gives optimal trajectories to all states on all stages during the cruise phase except points can not be reached. In order to prevent the aircraft climb or descent so frequently during the cruise phase, the simulation also set the aircraft should not perform another climb or descent if the distance between current location and previous climb or descent point is closer than 50 nmi.

6.3 Descent

While calculating for climb phase, the aircraft initial status and TOC altitude has been defined. Unlike climb, the calculation for descent is more complicated since both TOD altitude and position are not decided yet. With the result calculated by the optimization solver, the aircraft is possible to perform a descent from any point in the dynamic programming range.

Even though there are many options for TOD altitude and position, but the destination airport has already defined in the flight plan, the top of descent point of each altitude can be estimated by "rule of 3" which is introduced in appendix D. All states on the right side of the "rule of 3" line constitute the descent region. It is also the region where the aircraft descent mode should be triggered. The descent calculation should be performed on-by-one for all available TOD altitude. Its procedure is denoted by figure 16.

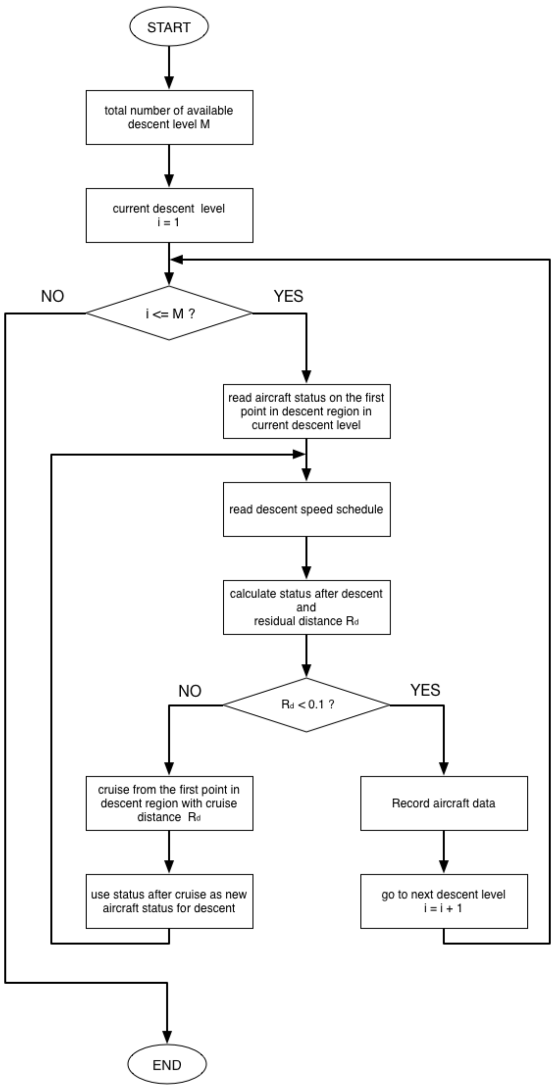


Figure 16: Diagram for descent calculation

With a selected TOD altitude, the initial descent point is assumed as the first point in descent region on this altitude. Perform a descent calculation start from the initial descent point, a residual distance between destination and descent end is calculated which denoted in figure 17. If the residual distance R_d is greater than 1 nmi, this distance will be used as residual cruise distance in the TOD altitude. The new aircraft descent status is recalculated after performing the residual cruise. The exact TOD position is calculated as mentioned above in a loop until the residual distance R_d smaller than 1 nmi.

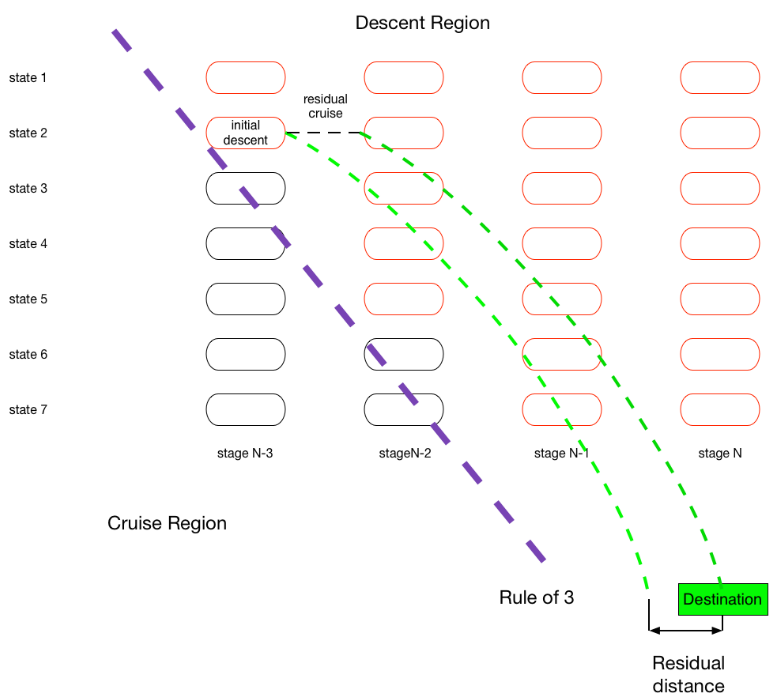


Figure 17: Descent calculation

6.4 Plot

The plot function generate a plot for optimal trajectory. Also, the headwind and tailwind are demonstrated with colored triangles in the plot. The darkness of the triangle's color shows the magnitude of wind.

Besides, two tables about en-route activities are generated as well. The first table gives aircraft speed at each waypoint. The second table gives positions of points where the aircraft change the cruising altitude.

7 Simulation Results

7.1 Simulation Instruction and Initialization

The following simulations and model verification calculations in the following chapter are performed by MATLAB R2017b. The MATLAB software was installed on an Apple MacBook pro which was produced in 2015 and equipped with a 2.9 GHz Intel Core i5 CPU and 8GB RAM.

To test the simulation model, establish a flight plan from Berlin in Germany (SXF/EDDB) to Stavanger in Norway (SVG/ENZV). The flight route and aircraft initial data are given in table 3.

Aircraft Type	B-737MAX8	Gross Weight [lb]	150,000	Cost Index	20
Flight Plan		Departure	EDDB	Destination	ENZV
		Cruise Level	FL330	Distance [nmi]	513
Flight Route	GERGA M725 KOGIM DCT SONAL M725 AAL L621 AMSEV Z324 RIVEX				

Table 3: Flight plan and aircraft initial data

7.2 General Results

With the flight route and aircraft data defined above, the optimal trajectory is computed and the following plot in figure 18. The weather data resolution for this simulation is pre-defined as 50 km. Colored Triangles implies wind direction and magnitude. The blue dash line marks all en-route waypoints. From the plot, the aircraft descends only once during cruise phase. A table contains information of the altitude changing point is generated.

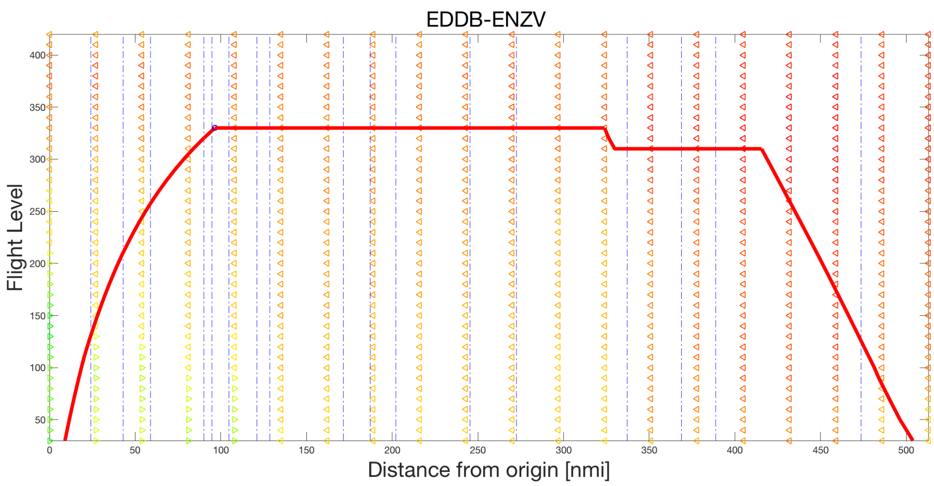


Figure 18: Optimal vertical trajectory from EDDB to ENZV with 50km weather data resolution

Position	Previous FL	New FL
AAL-13	330	310

Table 4: Altitude changing table

Where AAL-13 in table 4 means the cruising altitude changing point is 13 nmi before the waypoint called AAL. Besides, a table of aircraft speed and altitude on each way point is shown in table 5. This table includes way point name, aircraft altitude, speed magnitude and speed type.

Way Point	FL	Speed	Speed Type
MASOR	330	0.78	Mach
LABES	330	0.78	Mach
KOGIM	330	0.78	Mach
SONAL	330	0.78	Mach
CDA	330	0.78	Mach
BISTA	330	0.78	Mach
INPUN	330	0.78	Mach
ADSEN	330	0.78	Mach
AAL	310	0.78	Mach
LAGUM	310	284	CAS
AMSEV	310	284	CAS

Table 5: Speed table

7.3 Comparison

7.3.1 Compare with Highest Resolution Weather Data

With higher weather data resolution, run the simulation again for the same aircraft and same flight plan. A new trajectory is shown in figure 19 as well as the altitude changing point table and speed table correspond to the new trajectory.

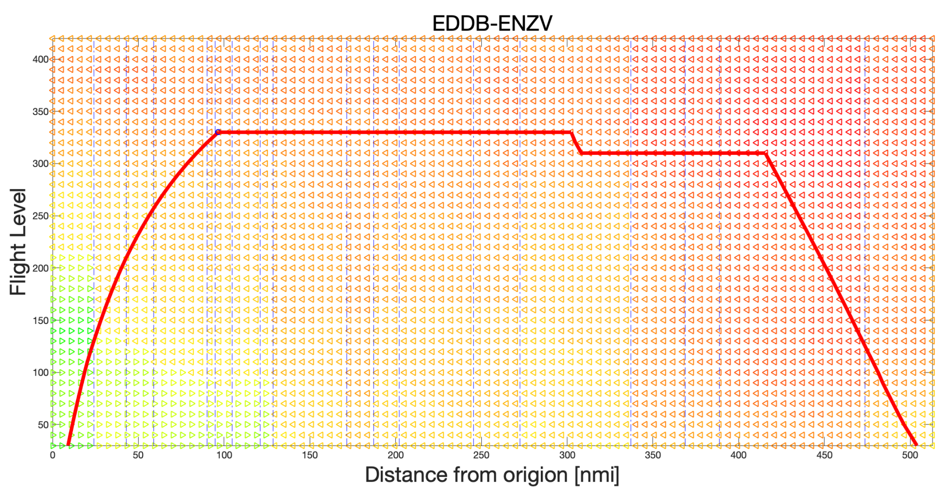


Figure 19: Optimal vertical trajectory from EDDB to ENZV with 10km weather data resolution

Position	Previous FL	New FL
AAL-34	330	310

Table 6: Altitude changing table

Way Point	FL	Speed	Speed Type
MASOR	330	0.78	Mach
LABES	330	0.78	Mach
KOGIM	330	0.78	Mach
SONAL	330	0.78	Mach
CDA	330	0.78	Mach
BISTA	330	0.78	Mach
INPUN	330	0.78	Mach
ADSEN	330	0.78	Mach
AAL	310	284	CAS
LAGUM	310	284	CAS
AMSEV	310	284	CAS

Table 7: Speed table

Flight trajectories shown in figure 18 and 19 are almost the same. Compare table 4 and table 6, the aircraft with higher weather data resolution descends to FL310 at the point 31 nmi before the other one. Status differences caused by different weather data resolution are denoted in table 8. From data in the table, the simulation with lower resolution weather data has better optimization result may caused by that the majority part during the route suffered from headwind, the weather data with lower resolution may skip some points that contains large magnitude headwind. But the percentage of each status difference caused by weather resolution is small and tolerable. The higher resolution data improves model precision but it also double the program execution time.

Resolution	50km	10km	Difference	Percentage
Fuel Consumption [lb]/[kg]	8,984/4,075	8,995/4,080	11/5	0.13 %
Flight Time [min]/[s]	76.90/4,614	77.00/4,620	0.10/6	0.12 %
Equivalent Fuel Consumption [lb]/[kg]	11,547/5,238	11,561/5,244	14/6	0.13 %
Program Execution Time [s]	25.70	57.57	31.87	55.36 %

Table 8: Status difference

7.3.2 Compare with Different Resolution Weather Data

In order to quantify model result precision caused by different weather data resolution, here define an deviation factor D as the ratio of the difference of aircraft status or program execution time for a model has weather data with a certain resolution over that from a model has highest resolution weather data. For example, the deviation factor for fuel consumption of the model with 50 km weather data resolution is,

$$D_{f_{50}} = \frac{W_{f_{50}} - W_{f_{10}}}{W_{f_{10}}} \tag{36}$$

Resolution	Fuel Consumption [lb]/[kg]	Deviation Factor
10 km	8,995/4,080	0 %
20 km	8,988/4,076	-0.08 %
30 km	8,991/4,078	-0.04 %
50 km	8,984/4,075	-0.13 %
80 km	8,964/4,066	-0.35 %
100 km	8,945/4,057	-0.55 %
140 km	8,942/4,056	-0.59 %
Resolution	Flight Time [min]/[s]	Deviation Factor
10 km	77.00/4,620	0 %
20 km	76.93/4,616	-0.09 %
30 km	76.97/4,618	-0.04 %
50 km	76.90/4,614	-0.12 %
80 km	76.70/4,602	-0.38 %
100 km	76.49/4,589	-0.65 %
140 km	76.48/4,589	-0.67 %
Resolution	Equivalent Fuel Consumption [lb]/[kg]	Deviation Factor
10 km	11,561/5,244	0 %
20 km	11,552/5,240	-0.08 %
30 km	11,557/5,242	-0.04 %
50 km	11,547/5,238	-0.13 %
80 km	11,521/5,226	-0.35 %
100 km	11,495/5,214	-0.57 %
140 km	11,491/5,212	-0.61 %
Resolution	Program Execution Time [s]	Deviation Factor
10 km	57.57	0 %
20 km	33.42	-41.95 %
30 km	31.78	-44.80 %
50 km	27.67	-51.94 %
80 km	26.49	-53.99 %
100 km	24.63	-57.22 %
140 km	24.14	-58.07 %

Table 9: Status for model with different weather data resolution

Figure 20 shows effectiveness factors for aircraft status calculated from model with different resolution weather data. For models with higher weather data resolution, aircraft status calculated from those models are more close too the base line which established with the simulation has highest resolution weather data. While decrease model resolution, the simulation results start to deviate from the base line. Due to lack of sufficient weather information, the model with lower resolution weather data leads to imprecise results.

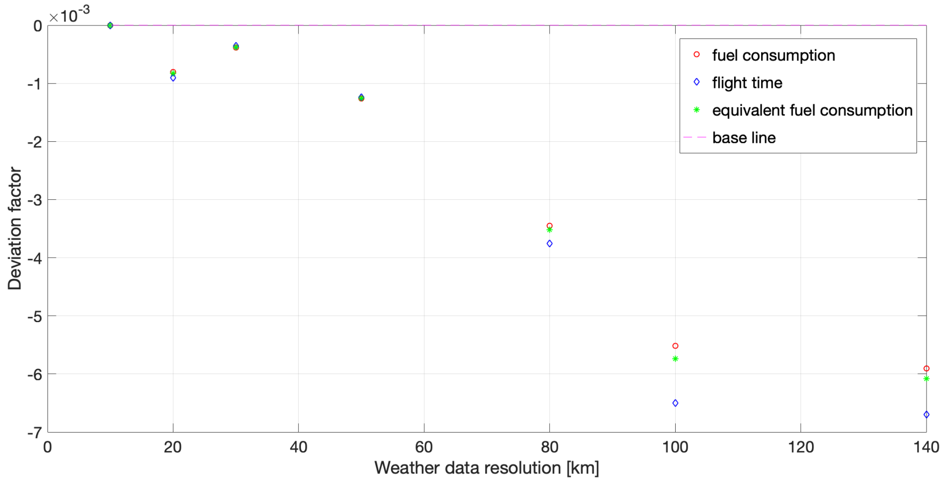


Figure 20: Deviation factors

The simulation model execution time is denoted in figure 21. At the beginning, the program running time decrease dramatically while decrease weather data resolution. But for models with weather data resolution worse than 50 km, the execution time dose not decrease too much. Combine with information showed in figure 20, the weather data with 50 km resolution can guarantee both model precision and program execution time at the same time.

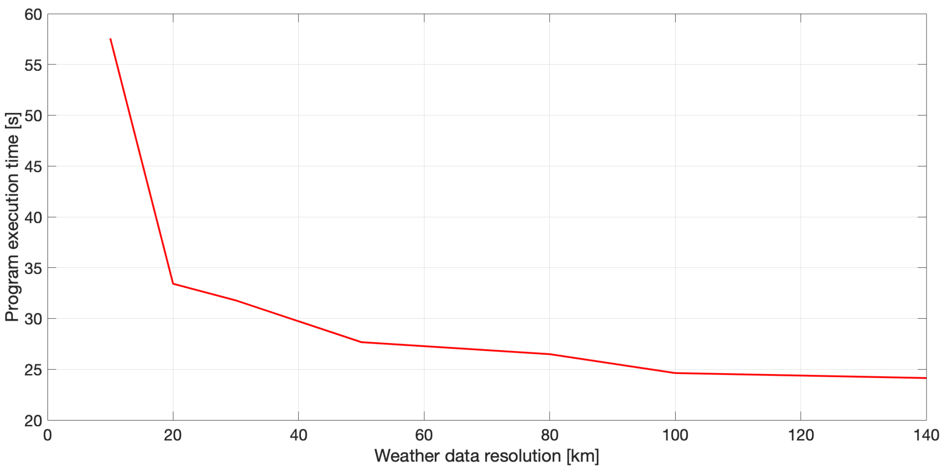


Figure 21: Program execution time for models with different resolution weather data

7.3.3 Compare with Fixed Level Cruise

For such a short flight defined in chapter 7, it has high probability to cruise in a fixed flight level due to ATC or weather constrains. In the simulation model, it is possible to force the aircraft cruise in a fixed flight level or avoids some levels in the route. Based on calculations with 50 km resolution weather data, the table below compares status computed from a fixed cruise level flight and optimized aircraft status.

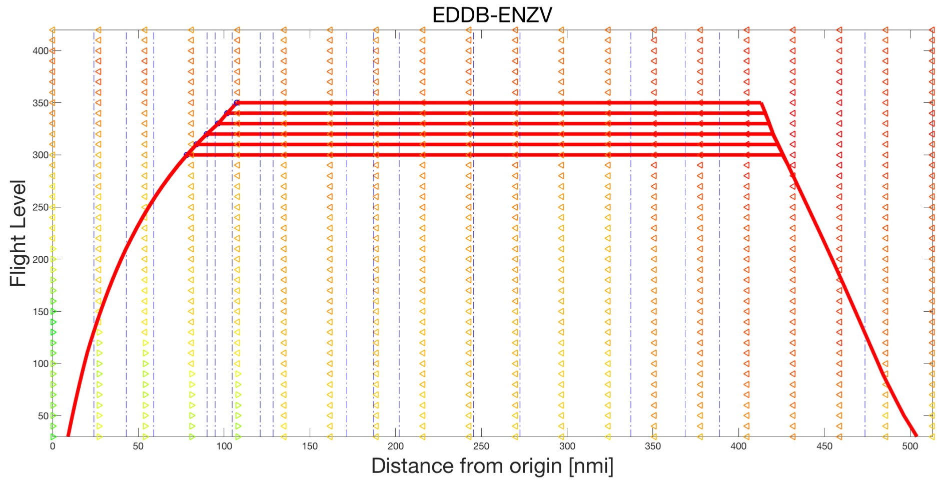


Figure 22: Fixed level cruise from EDDB to ENZV

Status	Fuel Consumption [lb]/[kg]	Difference [lb]/[kg]	Percentage
Simulation	8,984/4,075	-	-
FL300	9,008/4,086	24/11	0.27 %
FL310	8,996/4,081	12/6	0.15 %
FL320	9,049/4,104	65/29	0.71 %
FL330	9,110/4,132	126/57	1.40 %
FL340	9,148/4,149	164/74	1.84 %
FL350	9,182/4,165	198/90	2.21 %
Average	9,082/4,120	98/45	1.18 %
Status	Flight Time [min]/[s]	Difference [min]/[s]	Percentage
Simulation	76.90/4,614	-	-
FL300	77.27/4,636	0.37/22	0.48 %
FL310	76.85/4,611	-0.05/-3	-0.07 %
FL320	78.23/4,644	1.33/30	0.65 %
FL330	78.30/4,698	1.40/84	1.82 %
FL340	78.50/4,710	1.60/96	2.08 %
FL350	78.71/4,722	1.81/108	2.34 %
Average	77.98/4,679	1.41/65	1.41 %
Status	Equivalent Fuel Consumption [lb]/[kg]	Difference [lb]/[kg]	Percentage
Simulation	11,547/5,238	-	-
FL300	11,583/5,254	36/16	0.31 %
FL310	11,557/5.242	10/4	0.08 %
FL320	11,656/5,287	109/49	0.94 %
FL330	11,720/5,316	146/78	1.50 %
FL340	11,765/5,336	191/98	1.87 %
FL350	11,806/5,355	259/117	2.23 %
Average	11,681/5,298	134/60	1.15 %

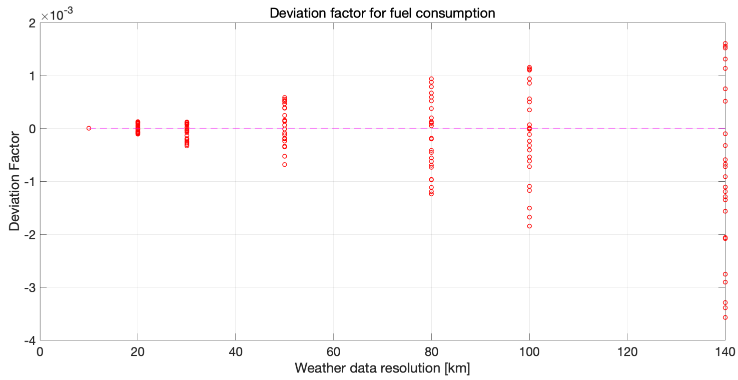
Table 10: Status for fixed cruise altitude flights

From data in table 10, with the same resolution weather data, the simulation result saves both fuel and time compare to fixed level flights.

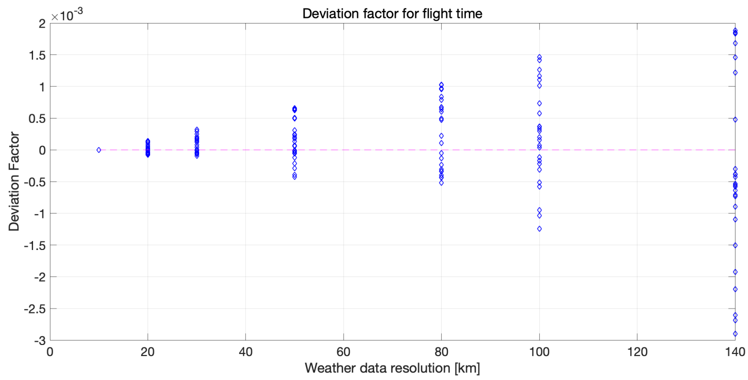
8 Weather Data Resolution and Model Sensitivity

With the deviation factor defined in section 7.3.2, more weather data is generated to test the relationship between weather data resolution and model sensitivity.

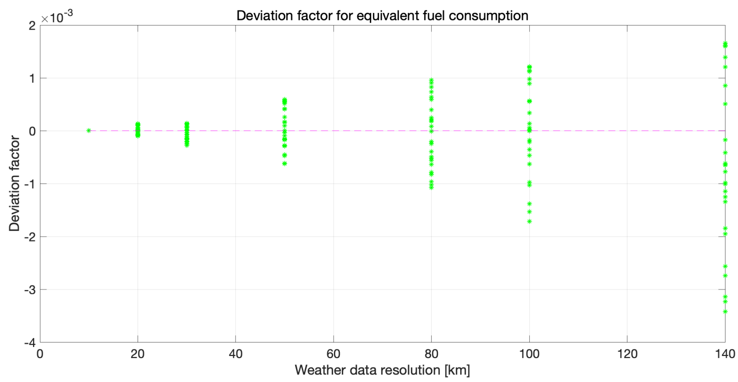
The test route is the same as the route mentioned in section 7, but the weather data was from 14/DEC/2018 UTC 16.00 to 15/DEC/2018 UTC 16.00 with one hour time step. By calculate deviation factors for those 25 groups of weather data, the distribution for deviation factors for this specific day are displayed in following figures.



(a) Fuel consumption



(b) Flight time



(c) Equivalent fuel consumption

Figure 23: Distribution for deviation factors

Setting the results calculated with 10 km resolution weather data as the base line, the deviation factors for fuel consumption, flight time and equivalent fuel consumption for low resolution weather data within 24 hours are more or less evenly distributed on both positive and negative side of the base line. From figure 23, the simulation results start to deviate from the base line while increase the distance between two weather points. The simulations with 20 or 30 km weather data resolution have better results compare with the rest groups. But from previous tests those two groups with more precise results also come with longer calculation time. Even though the group with 50 km weather data resolution gives deviated result, the deviation factors can be controlled within $\pm 0.1\%$. From previous experience, further decrease of the weather data resolution does not bring too much benefit on model calculation time.

This sensitivity study also verifies the high resolution weather data produced by Met Office brings better simulation results than the weather data with 140 km resolution generated by traditional weather prediction model. Notice that this sensitivity study only applied with short flight with B737-MAX 8. For other longer flights or other aircraft type, similar tests should be done to select the best weather data resolution supplied to airlines.

9 Model Verification

9.1 Model Defect

From chapter 3.4, the speed for a stage depends on aircraft status on the stage. Which means the speed in current stage could be affected by previous choices. In other words, if the aircraft waste more fuel in previous stages, it has the potential to save some fuel in following stages. But the simulation shows that the equivalent fuel consumption difference caused by decisions made in previous stages is small enough to be neglected. The proof of this can be found in appendix E.

9.2 Verify with Traversal Method

Theoretically, each path from the first to the last stage is unique. The optimal minimum equivalent fuel consumption should be calculated by traversal method which increases calculation time exponentially. To verify the model, a $5{\times}5$ reduced simulation model is formed as shown in figure 24. The model has only 7 weather points and the distance between each weather point is almost 150 km. The aircraft has initial gross weight $W = 150,000$ lb at the initial state on the first weather point. Between initial state and end state there exist 5 stages with 5 available flight levels among each stage.

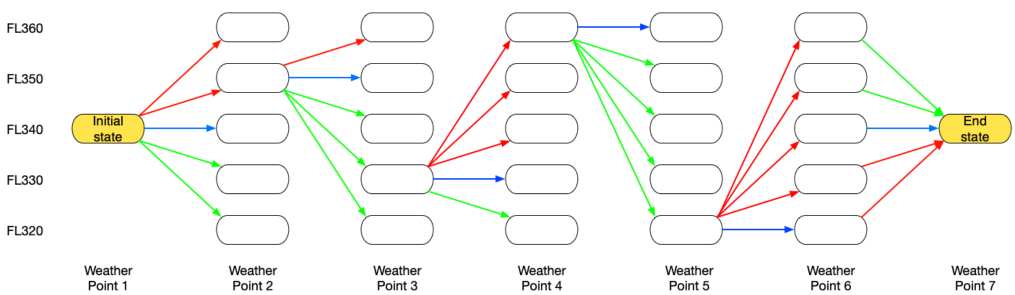


Figure 24: A $5{\times}5$ reduced weather data matrix

With such a long distance between two weather points, it is possible for the aircraft to move from any flight level on the current weather point to any flight level on the next weather point as denoted in figure 24. With weather data extracted from weather prediction model, the optimal results calculated by both methods are shown in table 11.

Method	Traversal method	DP method
Fuel consumption [lb]/[kg]	6,462/2,931	6,462/2,931
Flight time [min]/[s]	68.83/4,130	68.83/4,130
Equivalent fuel consumption [lb]/[kg]	8,756/3,972	8,756/3,972
Number of checked paths	$5 \times 5 \times 5 \times 5 \times 5 = 3125$	$5 + 5 \times 5 \times 4 + 5 = 110$
Program execution time [s]	212.52	1.50

Table 11: DP vs. Traversal method

The optimal trajectory calculated by dynamic programming is the same as the one calculated by traversal method. From traversal method point of view, the calculation go through all 3125 different options to move from initial state to the end state which causes program running time increase exponentially. This comparison between dynamic programming and traversal method also proof that the model defect mentioned in the previous section can be neglected.

9.3 Test with Extreme Weather

The simulation model supposed to supply aircraft with the wind-based en-route optimal trajectory. In other words, if a beneficial jet streams contains tailwind appears in the route, the simulation should be able to tell the aircraft to cruise at the altitude where has the jet stream. To verify this, based on the short flight defined in chapter 7, an extreme weather matrix is formed to test model result. The most part of the en-route wind is set to 0 with only one path contains beneficial tailwind left.

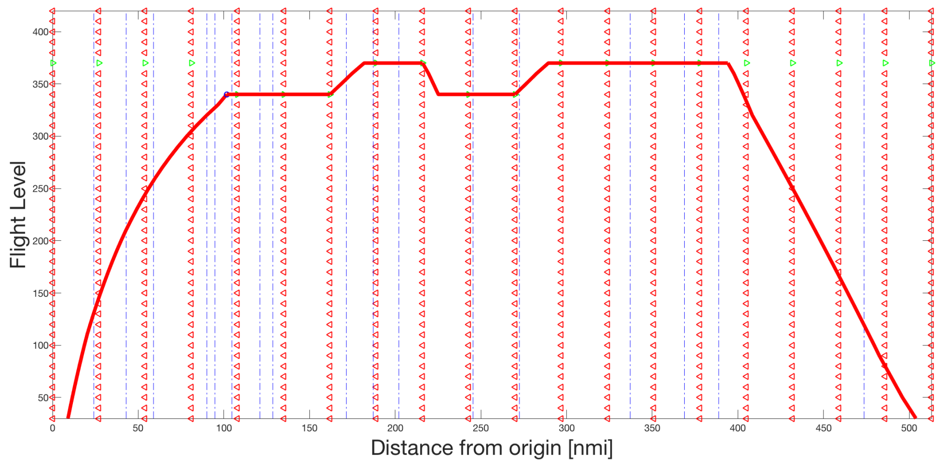


Figure 25: Fake weather data with 30 m/s tailwind

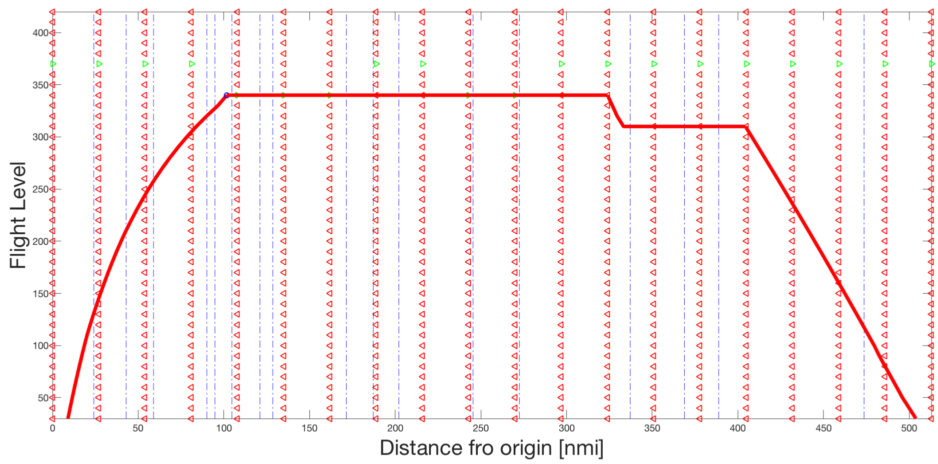


Figure 26: Fake weather data with 10 m/s tailwind

Figure 25 and 26 have a same path with tailwind which are shown with green arrows. The region with red arrows has has almost zero wind. In figure 25, the tailwind magnitude is 30 m/s. The simulation under this condition tells the aircraft to follow as more beneficial points as possible. But if the magnitude of tailwind decrease to 10 m/s as in figure 26, the aircraft will not climb to FL370 to follow points with beneficial tailwind. The simulation result implies that climb to a higher flight level will cause more fuel consumption.

9.4 Further Tests with Airlines

The aircraft performance data for the current model was extracted from EUROCONTROL's BADA model, further tests will be held between AVTECH Sweden AB and its cooperating airlines. These tests will use real aircraft performance data from aircraft manufacturers and aircraft operating history.

10 Model Extension

10.1 Aircraft Type and Constrains

The aircraft and engine performance data for B-737 MAX 8 used in the simulation was extracted from BADA model and aircraft operating limits were converted from charts in the aircraft operating manual. Similar, with enough aircraft and engine data, and some modifications for aircraft operating limits, the simulation model has the potential to be applied on most of commercial aircraft.

The constrains given in the model now are more focus on ATC constrains. It is now a simple matrix with elements inputted by pilots. It could be more dynamic which collect real-time ATC contains, hazard weather constrains and turbulence information. The simulation will be able to automatically avoid severe turbulence that may damage the aircraft.

10.2 Optimization Objective

The objective equation for the trajectory optimization used in the simulation is equivalent fuel consumption which shows some trade-off between time and fuel consumption. Theoretically, all aircraft status can be treated as objective equation.

10.3 Speed Change Through Dynamic Cost Index

Under ECON mode, the aircraft speed is only related with the cost index. It is possible to use reverse engineering technologies to analysis a reverse interpolation function to calculate cost index. With an optimized vertical trajectory with fixed cost index, if the cost index could varies based real weather data as well, it is possible to save more fuel.

11 Nomenclature

ATC	Air Traffic Control	
FMS	Flight Management computer	
EFB	Electronic Flight Bag	
LTO	Landing and Take-Off	
IAS	Indicated Airspeed	
CAS	Calibrated Airspeed	
TAS	True Airspeed	
GS	Ground Speed	
M	Mach number	
KIAS	Indicated Airspeed in Knots	
KCAS	Calibrated Airspeed in Knots	
TOC	Top of Climb	
TOD	Top of Descent	
MRC	Maximum Range Cruise	
LRC	Long Range Cruise	
VNAV	Vertical Navigation	
ISA	International Standard Atmosphere	
CDU	Control Display Unit	
C	Cost	[$]
C_f	Fuel related cost	[cents/lb]
C_t	Time related cost	[$/h]
C_{fix}	Fixed cost	[$]
F	Fuel consumption	[lb]
T	Flight time	[h]
CF	Cost Function	[lb]
CI	Cost Index	[100h/lb]
CCI	Corrected Cost Index	[100h/lb]
CCF	Corrected Cost Function	[100h/lb]
$CCIW$	Wind-adjusted Cost Index Value	[100h/lb]
W	Weight	[lb]
M	Mach number	
V_T	True airspeed	[m/s]
V_{TAS}	True airspeed magnitude	[m/s]
V_G	Ground speed	[m/s]
V_W	Wind speed	[m/s]
V_{HW}	Head/tail-wind speed	[m/s]
V_{CW}	Crosswind speed	[m/s]
RoC	Rate of Climb	[fpm]
RoD	Rate of Descent	[fpm]
FF	Fuel Flow	[kg/h]
t	Time	[h]
$Range$	Horizontal distance	[nmi]
δ	Atmospheric pressure ratio	
θ	Atmospheric temperature ratio	
D	Deviation factor	

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A Airspeed

IAS (Indicated Airspeed) is the speed converted from impact pressure ΔP. The impact is measured as the difference of total pressure P_t and static pressure P_s measured directly by Pitot-static probe [18]. For aircraft speed below than 200 knots, the impact pressure is considered equal to dynamic pressure.

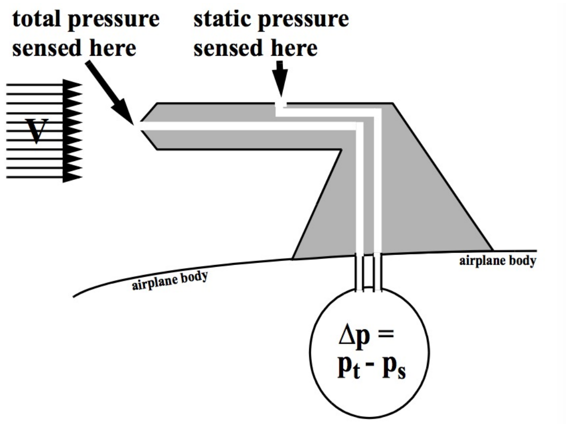

Figure 27: Simplifed diagram of a Pitot-static probe

CAS (Calibrated airspeed) is the airspeed calculated by electronic airspeed indicator. For aircrafts with only mechanical airspeed indicator such as Pitot-static probe, the indicated airspeed that has been adjusted of instrument errors can be called calibrated airspeed as well.

TAS (True Airspeed) is the aircraft speed respect to air. This speed is a correction of the calibrated airspeed based on real temperature.

GS (Ground Speed) is the aircraft speed respect to the ground. This speed is calculated by vector summation of true airspeed and wind speed. The ground speed can be also calculated by on board inertia navigation system or GPS.

B Crossover Altitude

The crossover altitude is defined as the altitude at which the specified CAS becomes the specified Mach number. At this altitude, the TAS converted from CAS equal to that converted from Mach number. The crossover altitude can be calculated by the formulas below.

Physical constants:
ISA constants at sea level (SL): pressure P_0, temperature T_0, speed of sound a_0 and density ρ_0
Gas constant for air: $R = 287.053 \quad [J/Kg{\cdot}K]$
Specific heat for air: $\gamma = 1.40$
gravitational acceleration: $g = 9.80665 \quad [m/s^2]$
temperature lapse rate: $\lambda = -0.0065 \quad [K/m]$

With given Calibrated airspeed V_{CAS} and Mach number M, the pressure ratio at transition altitude is,

$$\delta_{trans} = \frac{[1 + (\frac{\gamma-1}{2})(\frac{V_{CAS}}{a_0})^2]^{\frac{\gamma}{\gamma-1}} - 1}{[1 + \frac{\gamma-1}{2}M^2]^{\frac{\gamma}{\gamma-1}} - 1} \tag{37}$$

Then the temperature ratio at transition altitude can be calculated as,

$$\theta_{trans} = (\delta_{trans})^{-\frac{\lambda}{g}} \tag{38}$$

The crossover altitude is,

$$H_{trans} = [\frac{1000}{(0.3048) \times (6.5)}]{\cdot}[(T_0 + \Delta T){\cdot}(1 - \theta_{trans})] \tag{39}$$

where ΔT in equation 39 is the ISA deviation at sea level.

This method mentioned above is a rough estimation of crossover altitude. More realistic, FMS do not calculate crossover altitude directly with this method due to the limitation of computer power. In stead, it compares the true airspeed converted from both calibrated airspeed and Mach number. At lower altitude there exist $V_{CAS} < V_M$, at the moment that $V_{CAS} = V_M$, the FMC will automatically trigger a mode change.

C Speed Conversion

Use same physical constants in appendix B.

At a decided conversion pressure altitude, pressure P can be read directly from ISA table. With the real temperature T, air density ρ and speed of sound a are,

$$\rho = \frac{P}{RT} \tag{40}$$

$$a = \sqrt{\gamma RT} \tag{41}$$

For CAS/TAS conversion,

$$V_{TAS} = \sqrt{\frac{2}{\mu}\frac{P}{\rho}\left[\left(1 + \frac{P_0}{P}[(1 + \frac{\mu}{2}\frac{\rho_0}{P_0}V_{CAS}^2)^{\frac{1}{\mu}} - 1]\right)^{\mu} - 1\right]} \tag{42}$$

similarly,

$$V_{CAS} = \sqrt{\frac{2}{\mu}\frac{P_0}{\rho_0}\left[\left(1 + \frac{P}{P_0}[(1 + \frac{\mu}{2}\frac{\rho}{P}V_{TAS}^2)^{\frac{1}{\mu}} - 1]\right)^{\mu} - 1\right]} \tag{43}$$

where μ in both equation 42 and 43 is a ratio of γ.

$$\mu = \frac{\gamma - 1}{\gamma} \tag{44}$$

The Mach/TAS conversion is more simple, that is,

$$V_{TAS} = M{\cdot}a \tag{45}$$

D Top of Descent Calculation

In aviation, the descent rate is usually described with angle. In order to perform a slow, steady and comfortable descent for passengers, the aircraft normally should follow a descent angle of $3°$. This is also known as "rule of 3" or "$3:1$ rule of descent". In other words, when the aircraft descent for $1,000$ft, the horizontal distance for it travel through should be 3 nmi.

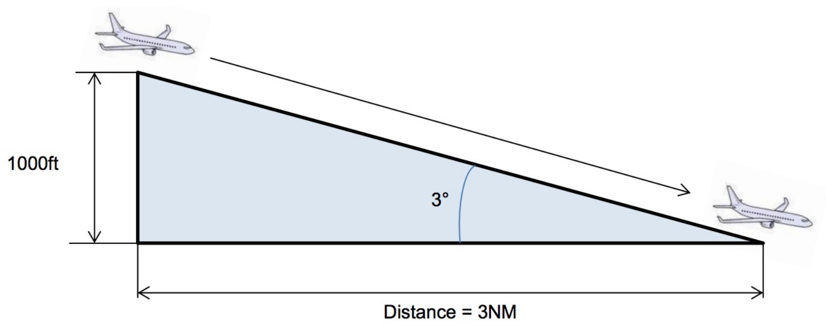

Figure 28: Rule of three

For a given descent angle $A_{descent}$, the descent rate can be calculated as,

$$R_{descent}(\%) = tan(A_{descent}) \tag{46}$$

The top of descent position can be estimated proximately as,

$$TOD_{distance} = \frac{FL_{cruise} - FL_{approach}}{A_{descent}} \tag{47}$$

Where $FL_{approach}$ in equation 47 is the flight level where the aircraft starts for final approaching. In the simulation, it is set at $3,000$ft (FL30). The method mention above only suitable for TOD position estimation.

E Simulation Data

Assume an aircraft with initial gross weight $150,000$ lb and cost index 15. When the aircraft reaches to the predefined TOC altitude (point A) at FL330 with aircraft status shows in table 12. Then the aircraft fly from A to C from different path denoted in figure 29. Through path 1, the aircraft first climb from FL330 to FL340 and reach to point D via point C. In path 2, the aircraft first cruise to point B and then climb to FL340. And it continues cruise in this flight level until point D. Points ABCD are connecting weather points and the distance between two nearby points is 27 nmi (about 50 km).

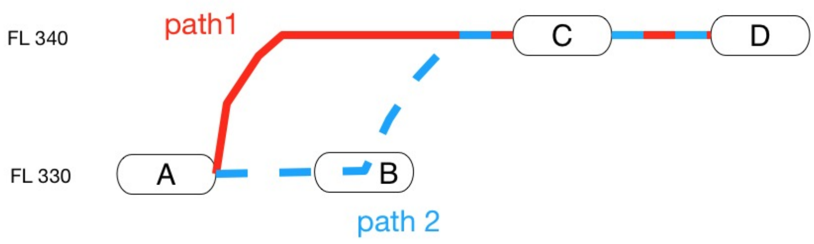

Figure 29: Different path

point	path	gross weight W [lb]	fuel W_f [lb]	time [h]	equivalent fuel W_{eq} [lb]
A	-	$147,025.5$	2975.5	0.2616	3367.9

Table 12: Aircraft status on point A

The status of the aircraft at point C via different path are shown in table 13. Obviously, the dynamic programming method will choose and record path 1. However, remember that lower aircraft weight can potentially save some fuel in later stage from point C to D.

point	path	gross weight W [lb]	fuel W_f [lb]	time [h]	equivalent fuel W_{eq} [lb]
C	1	$146,298.2$	3701.8	0.3888	4284.9
	2	$146.297.2$	3702.8	0.3890	4286.3

Table 13: Aircraft status on point C

Table 14 gives fuel consumption, time and equivalent fuel consumption from point C to D with different initial aircraft status on point C. The aircraft fuel consumption difference at point C is 1.4 lb, and fuel consumption saving with lighter aircraft is only 0.0024 lb.

point	path	fuel W_f [lb]	time [h]	equivalent fuel W_{eq} [lb]
D	1	346.7566	0.0635	442.0270
	2	346.7542	0.0635	442.0246

Table 14: Aircraft status on point D

The estimation time for the lighter aircraft from path 2 should continuous fly in order to reaches evenly equivalent fuel cost as the heavier aircraft from path 1 is,

$$time = (1.4/0.0024)\times 0.0635 \simeq 37 \quad [h] \tag{48}$$

Notice that the aircraft weight difference will decrease in later stage which means the potential fuel saving by lighter aircraft will decrease as well. The real time needed to reach the even point will be longer than 37 hours. Clearly, it would be unrealistic for commercial aircraft operated nowadays.

From the status comparison shown in previous tables, the dynamic programming method used in the simulation only record path 1 is enough for later stages optimization. As a conclusion, if the aircraft losses more fuel at early stages, it would be difficult for it to earn it back in later stages.

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